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FIRST REVISION

PAPER P-967

AN ANALYSIS AND COMPARISON OF THREE AIRCRAFT ATTRITION MODELS

PROBABILITY OF HIT BY ANTIAIRCRAFT GUNS

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SYSTEMS EVALUATION DIVISION

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AN ANALYSIS AND COMPARISON OF THREE AIRCRAFT ATTRITION MODELS

PROBABILITY OF HIT BY ANTIAIRCRAFT GUNS

J. A. Ross

July 1974

This report has been prepared by the Systems Evaluation Division of the Institute for Defense Analyses in response to the Weapons Systems Evaluation Group Task Order DAHC15 73 C 0200 T-182, dated 18 September 1972.

In the work under this Task Order, the Institute has been assisted by military personnel assigned by WSEG.

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PREFACE

This report was initially published in September 1973. Since that time it has been learned that some of the numbers presented were incorrect. Accordingly, the report has now been revised. It should be noted, however, that the models described are constantly being updated. Thus, statements contained herein that refer to the models as they existed a year ago may no longer be applicable.

The author wishes to express his appreciation for the assistance provided him in this study by Mr. Eugene F. Kelton (Naval Ordnance Laboratory, White Oak, Maryland), who did most of the work presented in Chapter V of this report, and by Mr. Bruce A. Morey (IDA), who did most of the work presented in Chapter VI.

Chapter I

INTRODUCTION AND SUMMARY

In recent years a number of studies and analyses have been conducted in which the expected attrition of U.S. aircraft by enemy antiaircraft guns was evaluated by means of mathematical computer models. Several different models are currently in use, but it is known that they do not agree with one another. Accordingly, a decision has been made to carry out a test and evaluation program to measure probability of hit on U.S. fixed-wing and rotary-wing aircraft fired on by enemy antiaircraft guns, when the conditions of engagement are known. This program, known as HITVAL, consists of three elements:

(1) An analysis and comparison of the models of interest.

(2) A joint field test with participation by both the Army and the Air Force.

(3) Validation and improvement of the models through analysis of the test data.

This report responds to the first of these elements. Its goals are as follows:

(1) To understand why the various models make predictions that do not agree with one another.

(2) To identify any obvious deficiencies that the models may have.

Additional goals requiring HITVAL test data for implementation are:

(1) To further identify deficiencies of the models.

(2) To indicate which of the models is the best; or, if none of them is deemed to be adequate, to determine the requirements to be satisfied by a new model.

The scope of this study is limited to engagements involving one aircraft and one gun. It is assumed that acquisition and identification of the target aircraft pose no problems whatsoever for the gun, and that the gun is not constrained by doctrine, logistics, or reliability. The models make no distinction between fixed- and rotary-wing aircraft, and hence (except where noted) none will be made here.

The models analyzed in this report are shown below.[1]

Model	*User*
EVADE	Army Materiel Systems Analysis Agency Aberdeen Proving Ground, Maryland
P001	Air Force Armament Laboratory Eglin Air Force Base, Florida
SIMFIND	Institute for Defense Analyses Arlington, Virginia

The approach taken in analyzing these models was to consider each of them as being composed of a number of components, or submodels–specifically, those performing the following functions: (1) interpolation of the flight path of the aircraft, (2) determination of the azimuth and elevation angles of the gun at the time of fire (i.e., determination of the direction of fire), (3) determination of the trajectory of the fired projectile, and (4) calculation of the probability of hit given the mean trajectory of the projectile and the flight path of the target aircraft. Comparison of the models was thus carried out by comparing these submodels. The results are summarized below:

- The heart of each model is its *gun angle determination submodel.* However, results obtained vary considerably among the models (see Table 1), and without HITVAL data it is impossible to say which (if any) of them are correct.
- The *flight path interpolation submodel* of EVADE may not be sufficiently accurate for analyses involving manueuvering fixed-wing aircraft.
- The *projectile trajectory determination submodels* of all three of the models are satisfactory.
- The *single-shot probability of hit calculation submodels* of all three of the models are satisfactory.[2]
- The investigation reported in this paper has not disclosed any basis for the elimination of any of these three models nor for the preference of any of these models. Selection of a preferred model cannot be accomplished without empirical data from the HITVAL field test.

1. Prior to the initiation of the HITVAL program, an extensive survey of air attrition models was conducted by the Model Comparison Working Group of the Joint Aircraft Attrition Program of the Joint Technical Coordinating Group for Munitions Effectiveness (JTCG/ME). The models recommended by JTCG/ME were examined for possible use in the HITVAL program, and the best of these were analyzed in detail as described in this report.

2. This statement assumes that the dispersion associated with the projectile has a bivariate normal distribution.

Chapter II of this report briefly describes the operation of each model, and Chapters III through VI discuss the analyses performed for the four submodels identified above.

Table 1. Encounter Hit Probabilities Obtained With Three Different Models

Flight Path	EVADE*	P001	SIMFIND
Gun System 1†			
Straight and Level**			
100-meter offset	.069	.039	.079
300-meter offset	.064	.041	.079
1,000-meter offset	.024	.017	.020
Bombing Path††	.092	.142	.078
Gun System 2†			
Straight and Level**			
100-meter offset	.030	.050	.032
300-meter offset	.041	.032	.049
1,000-meter offset	.028	.025	.031
Bombing Path††	.100	.166	.080
Gun System 3†			
Straight and Level**			
100-meter offset	.174	.392	.092
300-meter offset	.179	.454	.090
1,000-meter offset	.155	.510	.077
Bombing Path††	.064	.343	.059

Note: The differences in hit probability among the three models are due to differences in their gun angle determination submodels (and, in the case of the bombing flight path, to differences in their flight path interpolation submodels), but not to differences in any of the other submodels.

*Results f EVADE were provided by the Army Materiel Systems Analysis Agency.

†The three gun systems are the Soviet systems in the HITVAL test. A firing rate of one round per second was used in all the cases, and the hit probabilities presented represent the cumulative total for the entire encounter.

**Altitude 300 meters, speed 250 meters per second (485 knots).

††The bombing path is shown in Figure 1 (page 8).

Chapter II

SUMMARY DESCRIPTION OF THE MODELS

This chapter provides a brief description of each of the three models that are analyzed.[3] The operation of these models is basically as follows. The inputs are read in, and the target aircraft is moved along its flight path until it is within range of the gun. The inputs to the fire control system of the gun (i.e., the position and velocity of the aircraft) are then evaluated, the aimpoint is determined, and a projectile is fired. The probability of hit is now computed, and the whole process is repeated for the next round fired, until the aircraft is out of range.

In the real world, the inputs to the fire control system have errors associated with them. The chief differences among the models lie in the way these errors are simulated.

A. P001

P001 is an expected value model that evaluates only the means and variances of the errors associated with the inputs to the fire control system. These quantities in general are assumed to depend on the past and present position of the target, and on the time rate of change of this position. The user has no control over the values of these errors (other than by the specification of the gun type to be used), as the exact forms of the critical equations (as well as all the constants, coefficients, etc.) are completely fixed by the model. P001 then computes the aimpoint and calculates the resulting probability of hit by assuming that each of the errors is normally distributed. Once these calculations have been made, there is no need for any additional iterations to be performed.

B. SIMFIND

SIMFIND is a Monte Carlo model in which the values of the errors themselves (rather than just the means and variances) are determined. These values are functions of three things: (1) inputs (means and variances) specified by the user, (2) the previous values of the errors during the encounter, and (3) the values of numbers chosen at random. The complete

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3. Extremely detailed and complete descriptions of all three of the models already exist, and hence will not be presented here. These have been published under the auspices of the Advance Planning Group of the Joint Aircraft Attrition Program of the Joint Technical Coordinating Group for Munitions Effectiveness. Requests should be addressed to Mr. Hubert W. Drake, Naval Weapons Center, China Lake, California 93555.

encounter is simulated a specified number of times,[4] each time with a different random number sequence; the overall probability of hit for the entire engagement is taken to be the average over all the iterations.

C. EVADE

EVADE is a systematic sampling model. A number of iterations are performed for each round fired, and the single shot probability of hit is taken to be the average over all the iterations. The errors in the inputs to the fire control system of the gun do not depend on their previous values during the encounter, or on any random numbers. Instead, the user specifies the mean and variance of the error, and also the number of times it is to be systematically sampled. The value of the error for a given iteration is then determined internally by the program on the basis of these three specified quantities. The total number of iterations is equal to the product of N_1, N_2, N_3, ..., where N_j is the number of times the jth variable is to be systematically sampled.[5]

To give the reader some feeling for the complexity and capability of the three models, a delineation of the inputs they require (for a case involving only one aircraft and one gun—the only type considered in this study) is presented in Table 2. Additional discussion relating to the specific submodels is presented in the succeeding chapters.

4. Previous analyses have indicated 50 iterations to be adequate. Thus, all investigations performed in connection with this study used 50 iterations.

5. In this study N_j was always set equal to either one or three. The total number of iterations was 243 for gun systems 1 and 2, and 81 for gun system 3.

Table 2. Inputs Required by the Models

Inputs	*EVADE*	*P001*	*SIMFIND*
Firing rate of gun	x	x	x
Number of rounds per burst	x	x	x
Maximum range of gun	x	x	x
Maximum and minimum elevation angles of gun	x	x	x
Maximum slewing rates of gun	x	x	x
Resetting time required after attempt to exceed maximum slewing rates	x		x
Minimum time from acquisition to fire	x	x	x
Location of gun and fire director	x	x	x
Muzzle velocity:			
Mean	x	x	x
Standard deviation	x		x
Value assumed by fire control system	x		x
Drag constant(s) of projectile	x	x	x
Area of aircraft (front, side, bottom)	x	x	x
Flight profile of aircraft:			
Position	x	x	x
Velocity		x	
Attitude	x	x	
Number of iterations	x		x
Random number seed			x
Errors (mean and standard deviation) in inputs to fire control system	x	*	x
Tracking dispersion	x	*	x
Ballistic dispersion	x	*	x
Serial correlation time constant			x
Filter smoothing time constant		x	x
Gun type*	x	x	

*Evaluated internally by the model; no input required other than gun type.

Chapter III

FLIGHT PATH INTERPOLATION

Each of the various models interpolates flight paths differently. This is significant because differences in aircraft position and/or velocity generally lead to different theoretical aimpoints. The models all accept input flight path data in the form of discrete position points and associated times (P001 also requires the associated velocities). Interpolation between these points is performed as follows:

(1) EVADE–The velocity at the nth input point is taken to be the difference in position between input points n+1 and n, divided by the difference in time between these points.[6] Position and velocity are both interpolated linearly between input points.

(2) P001–Position and velocity are both interpolated linearly between input points, and are considered to be completely independent of one another.

(3) SIMFIND–Position is expressed as a polynomial in time, the coefficients being determined by a least squares error fit involving four consecutive points. Velocity and acceleration are determined by differentiation of the position with respect to time, and are both constrained to be continuous throughout the entire flight path.

Theoretically it is possible to attain any degree of accuracy simply by making the time interval between successive input points sufficiently small. It is thus appropriate to ask what the maximum allowable time interval is for each model consistent with the requirement that interpolation errors be small.[7] To answer this question, an investigation was carried out using as a standard an F-4 bombing pass flight path (Figure 1) generated by the Air Force Armament Laboratory (AFATL), Eglin Air Force Base, Florida; this flight path specified position and

6. Strictly speaking, EVADE requires the discrete aircraft position points and the average speed between successive points, but not the associated times. Motion between successive points is always assumed to be along the straight line segment connecting them. Under these conditions the difference in time between successive points is uniquely determined, and hence the procedure outlined in the text is completely equivalent to that actually carried out in EVADE.

7. *Feasibility of a Test of Probability of Hit by Antiaircraft Guns*, WSEG Report 190, August 1972, FOUO. Appendix D of R-190 derives constraints on the allowed size of the errors in aircraft position and velocity if the error in the theoretical aimpoint is not to exceed 1 mrad. These maximum allowed errors are (for fixed-wing aircraft):

Azimuth:	1 mrad	Course angle:	4 mrad
Elevation:	1 mrad	Climb angle:	4 mrad
Range:	0.4 %	Speed:	0.4 %

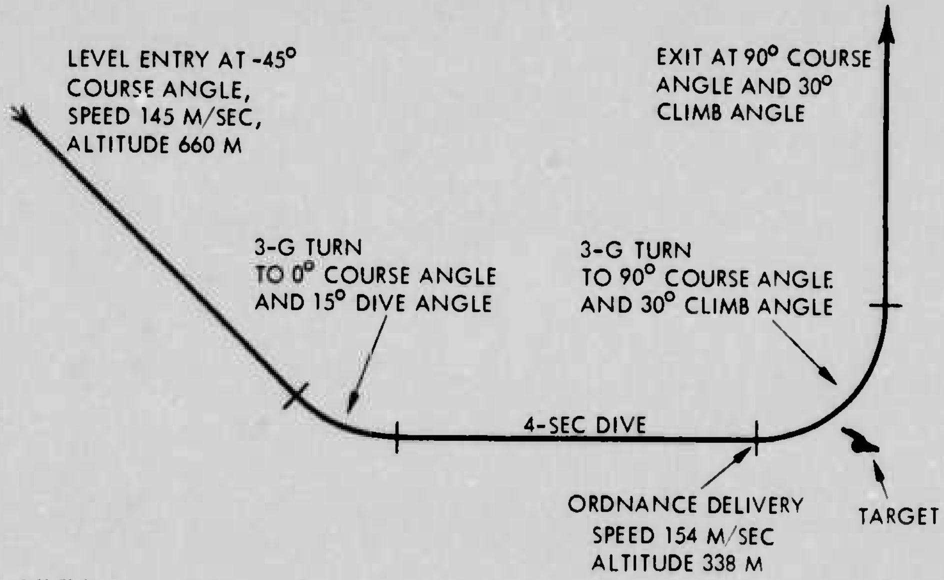

Figure 1. Bombing Flight Path

velocity every 0.2 second. The investigation consisted of giving each of the three models the first point of the path, and each nth point thereafter (e.g., if n = 5, the models would each be given points 1, 6, 11, 16, 21, 26, etc.). The predicted values of azimuth, elevation, range, course angle, climb angle, and speed were then compared every 0.2 second with those specified by the standard. The results (extrapolated where necessary) are presented in Table 3. It is thus apparent that for this particular example an extremely small time step is required to achieve high accuracy with EVADE.

*Table 3 Approximate Maximum Time Step**
(Seconds)

	EVADE	*P001*	*SIMFIND*
Azimuth	0.5	0.5	0.3
Elevation	0.8	0.8	0.5
Range	1.0	1.0	0.5
Course angle	0.02	0.5	0.3
Climb angle	0.05	0.5	0.3
Speed	0.10	1.0	0.5

*Consistent with error constraints derived in WSEG Report R-190.

Chapter IV

DETERMINATION OF THE GUN FIRING ANGLES

The heart of each of the models is the submodel that determines the azimuth and elevation angles of the gun at the time of fire (i.e., the direction of fire). These gun firing angles are computed on the basis of (1) the inputs to the fire control system of the gun, (2) linear extrapolation of the flight path of the target aircraft after fire, and (3) anticipated performance of the projectile to be fired. In order to compare the models, two separate investigations were conducted. The first used a straight and level flight path (so that all the models interpolated the path correctly), while the second used the AFATL flight path described in Chapter III of this report. The anticipated performance of the projectiles for a given gun system was the same throughout for all three models.[8] The results of these investigations, which were carried out for three different antiaircraft gun systems, are presented in Table I (Chapter I).

It is apparent that there is considerable variation among the models. In order to understand the reasons for this, it is helpful to examine the predictions that each model makes for the errors in the inputs to the fire control system of the gun. Figures 2 through 7 present the means and standard deviations of these variables as a function of time for gun system 1[9] for the straight and level flight path with 300-meter offset. As the figures indicate, the P001 procedures result in large errors and rapid changes at crossover (i.e., the point at which the gun-to-target distance is minimal), whereas in the other two models the errors tend to be relatively insensitive to time. For EVADE they are constant over the entire encounter (for a given iteration), whereas for SIMFIND—when the errors are not constant—the effects of autocorrelation and the vagaries of random number sequences combine to produce a relatively small variation, both in mean and standard deviation. Since the equations used by the P001 model involve a strong dependence on the time rate of change of target position with respect to the gun (while the equations used by the other two models do not), the wide variation in behavior exhibited by the models in Figures 2 through 7 is not too surprising.

8. Differences in hit probability were thus due to differences in the way the models simulate the inputs to the fire control system, but not to differences in projectile performance.

9. The procedures used in determining these errors have already been briefly described in Chapter II of this report.

Since the gun firing angles are determined by the inputs to the fire control system of the gun, one would expect wide variation among the models here as well. The means and standard deviations of the errors in these quantities[10] are presented for the above case in Figures 8 and 9. As expected, the errors depicted by the P001 model are large. For EVADE and SIMFIND, however, the mean errors are small compared to the standard deviations. The mean miss distances and single shot probabilities of hit are presented in Figures 10 and 11. Not surprisingly, the model with the largest errors in gun angles tends to have the largest mean miss distances and the smallest single shot hit probabilities.

In summary, the three models use different approaches to determine the aimpoint, with the result that there is a considerable lack of agreement among them as to the value of the probability of hit.

10.. Zero error refers to the gun firing angles that produce a direct hit on the target.

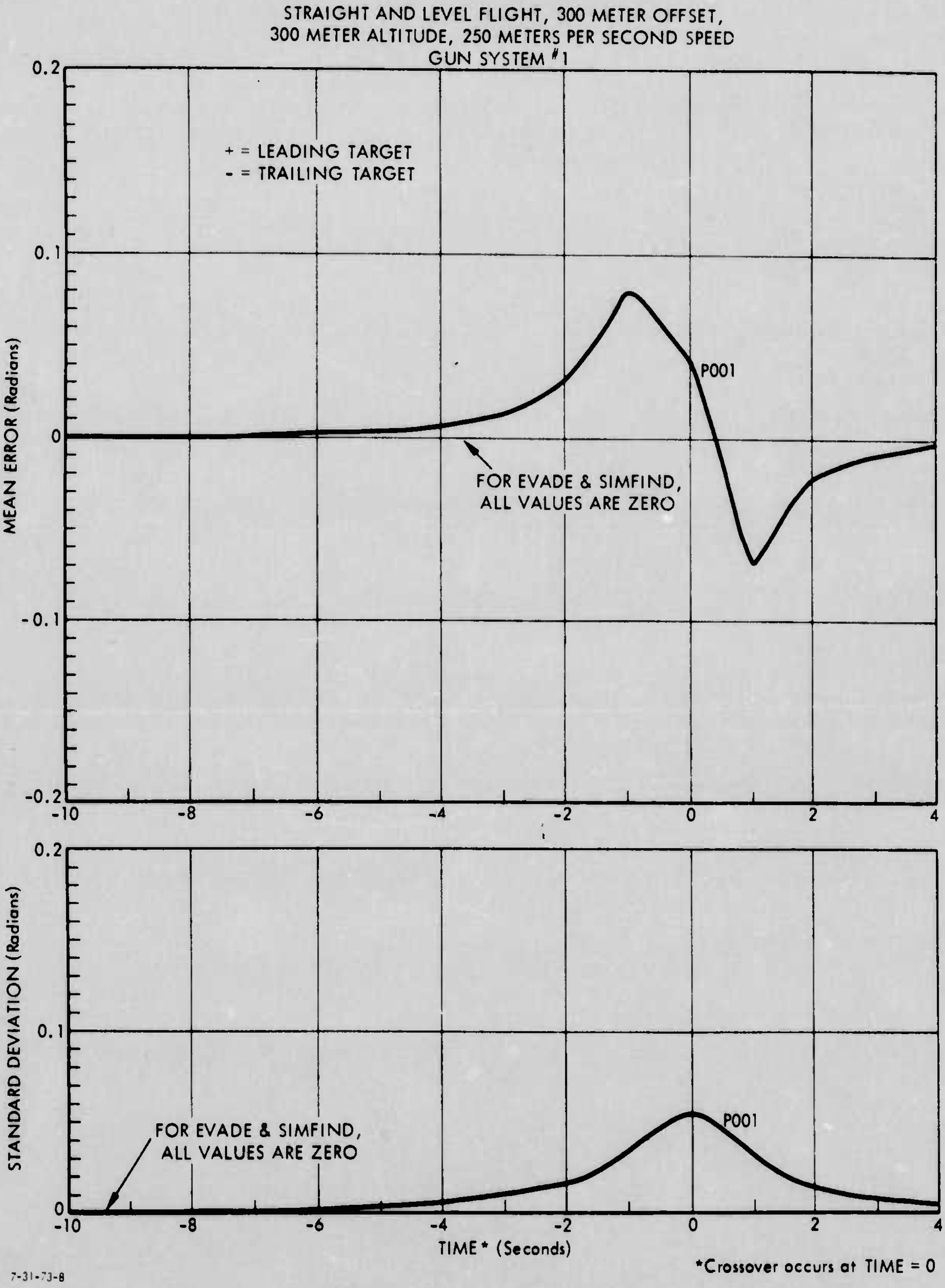

Figure 2. Errors in Azimuth

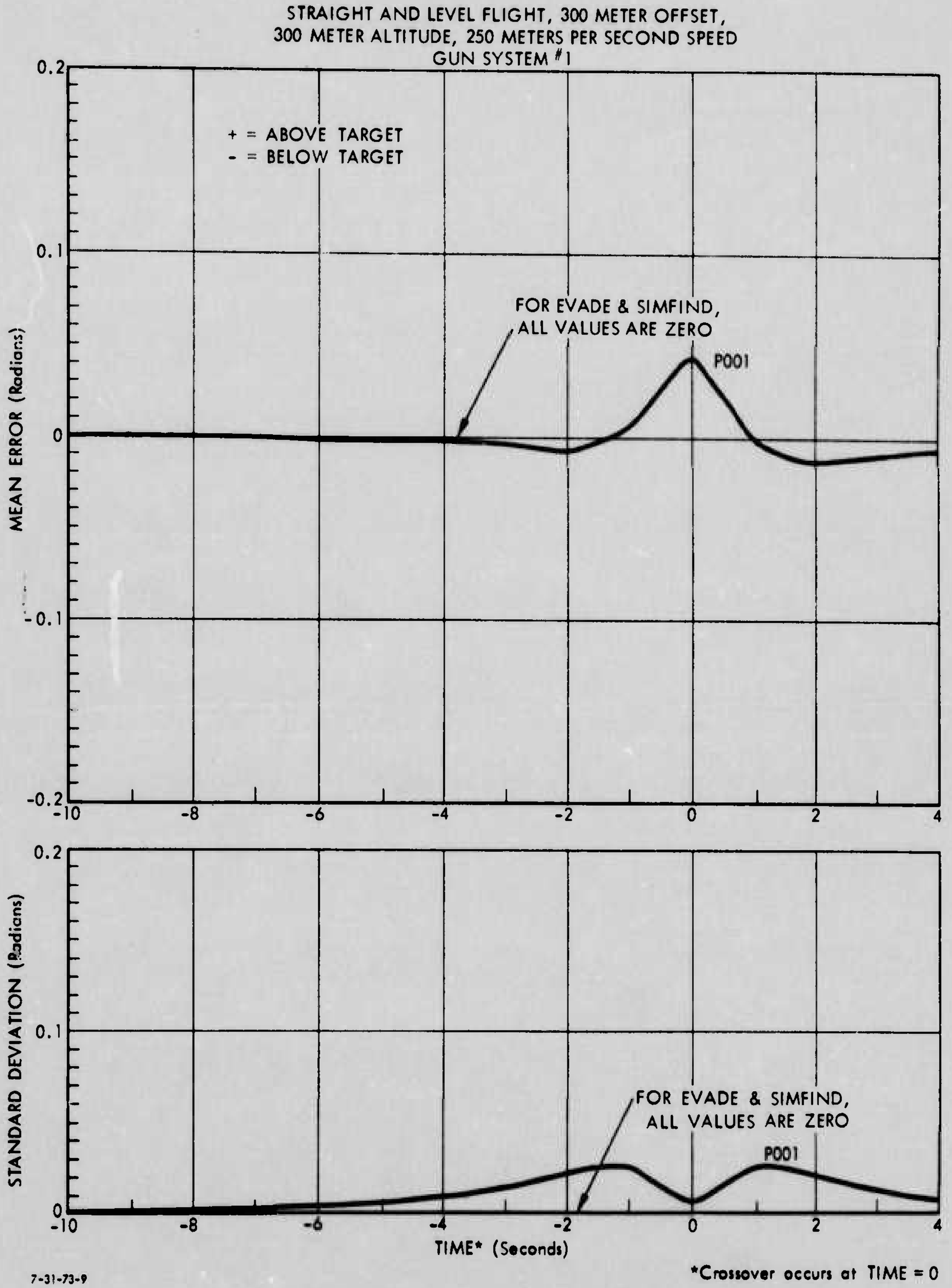

Figure 3. Errors in Elevation

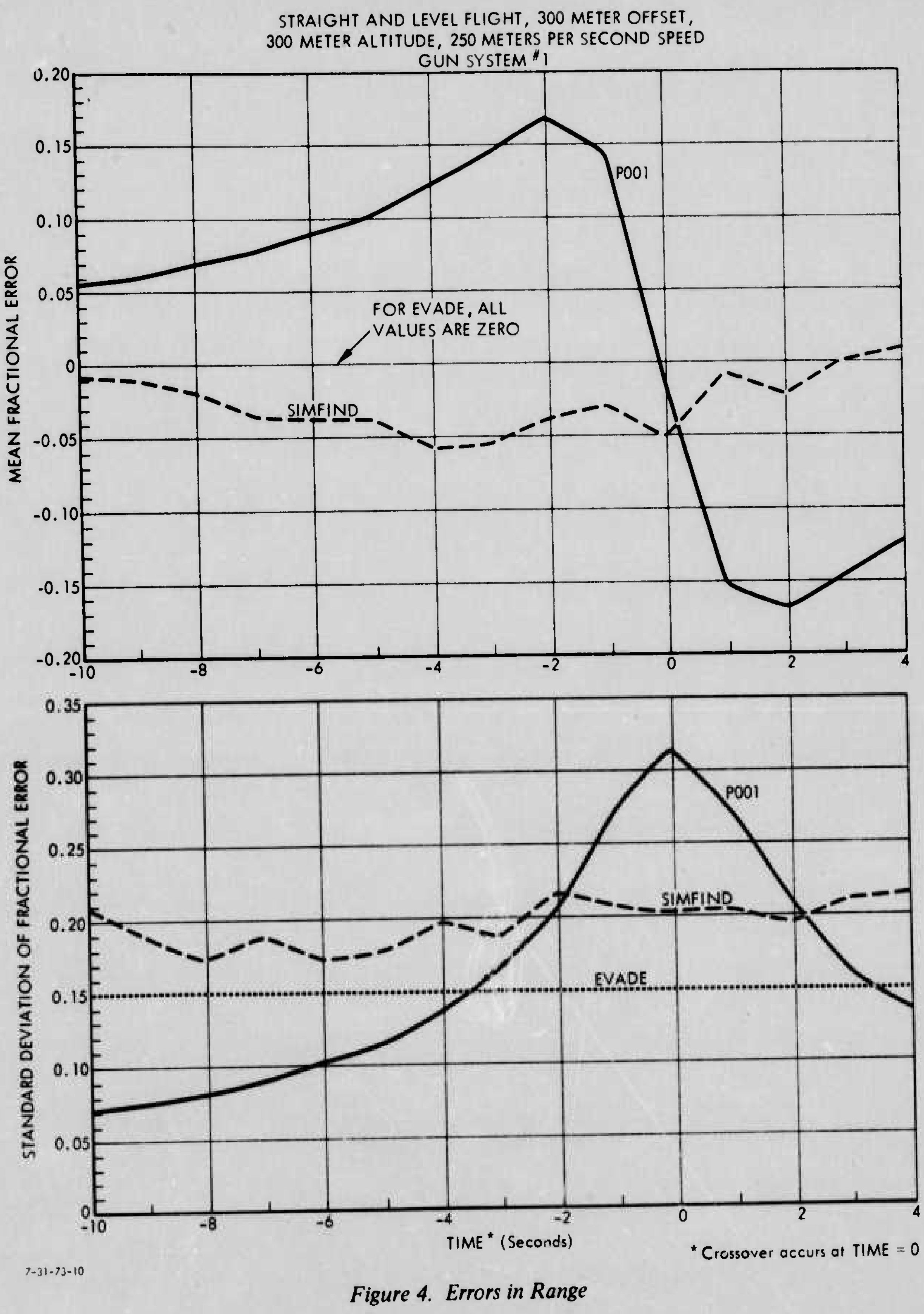

Figure 4. Errors in Range

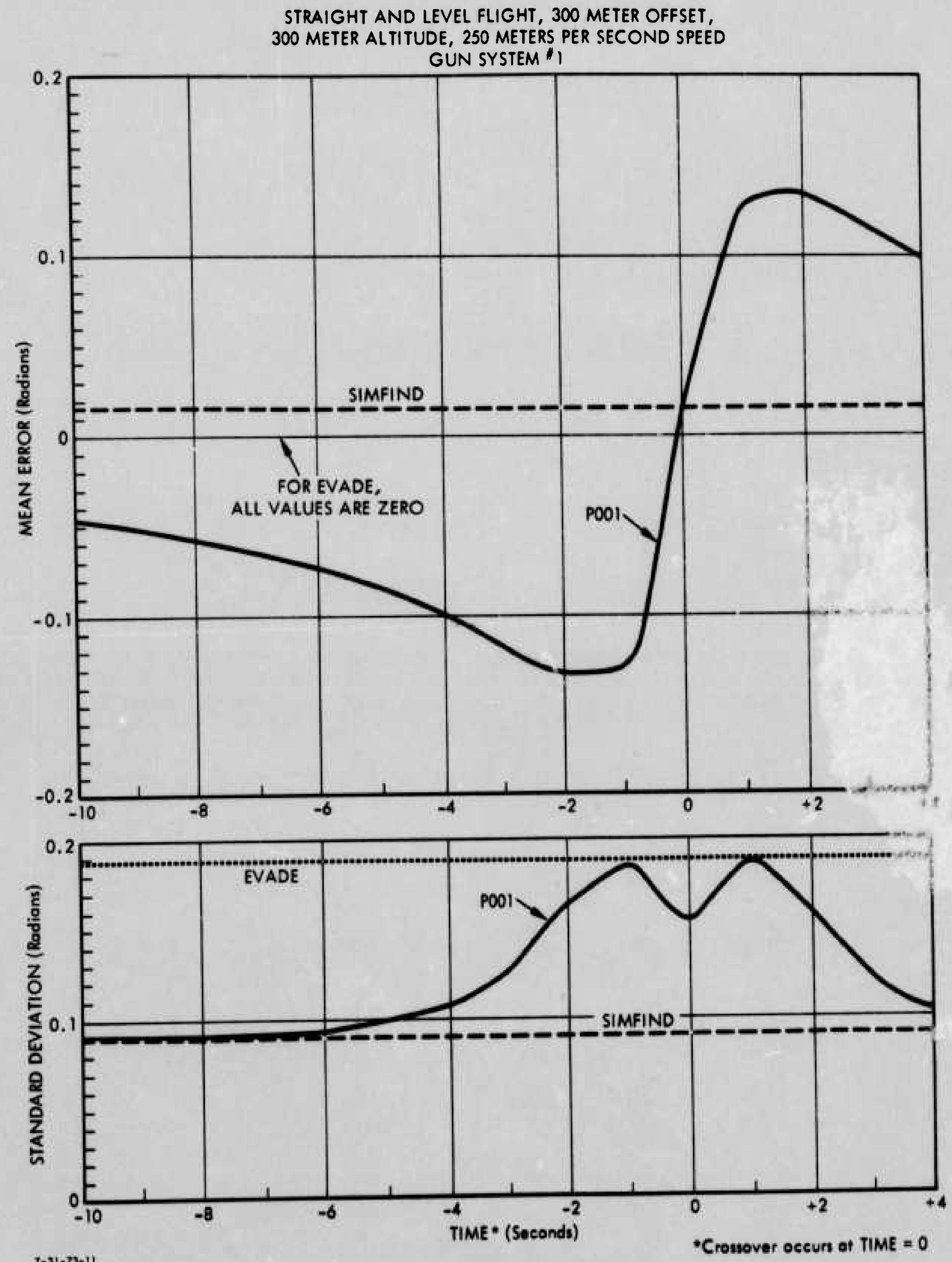

Figure 5. Errors in Course Angle

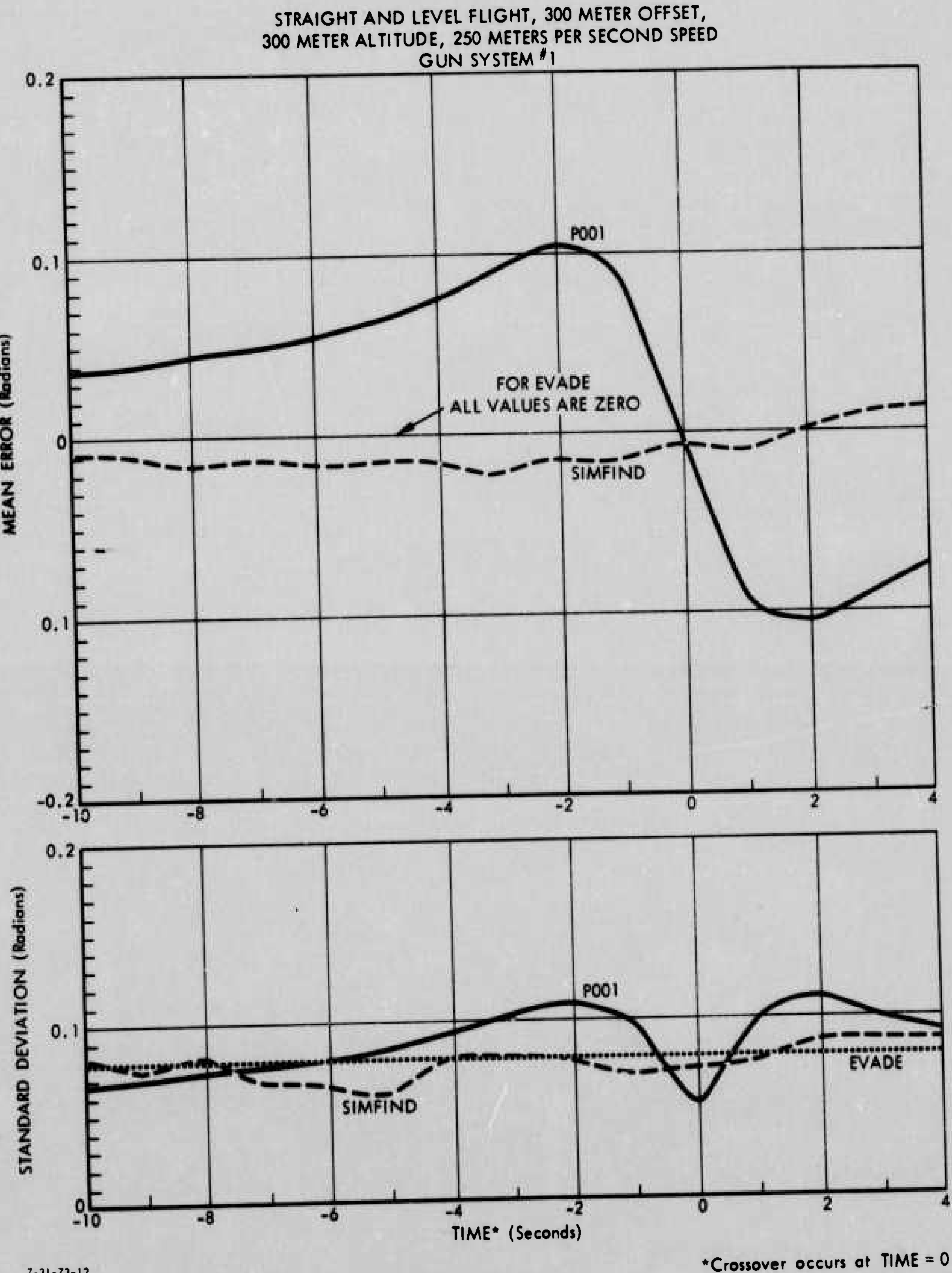

Figure 6. Errors in Climb Angle

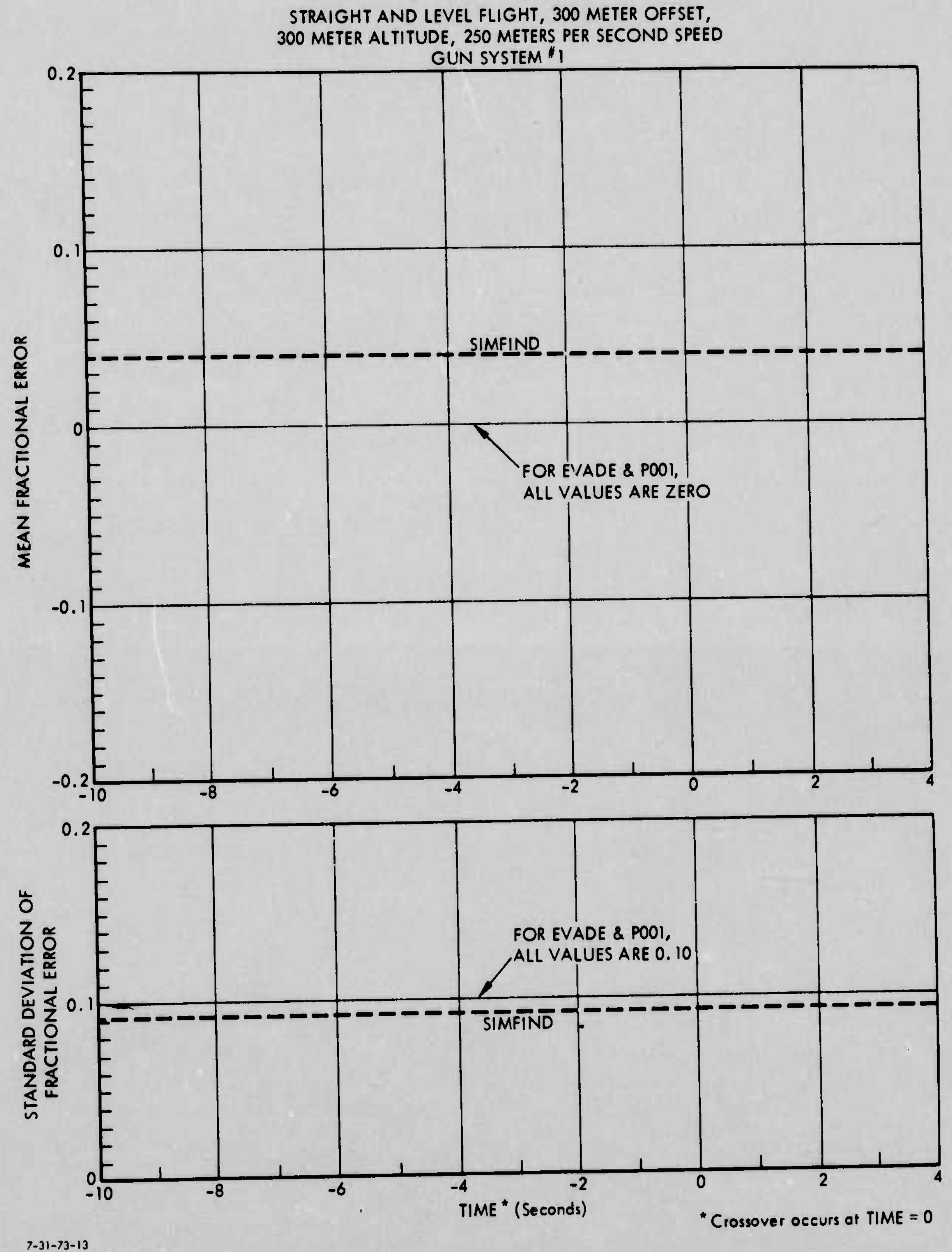

Figure 7. Errors in Speed

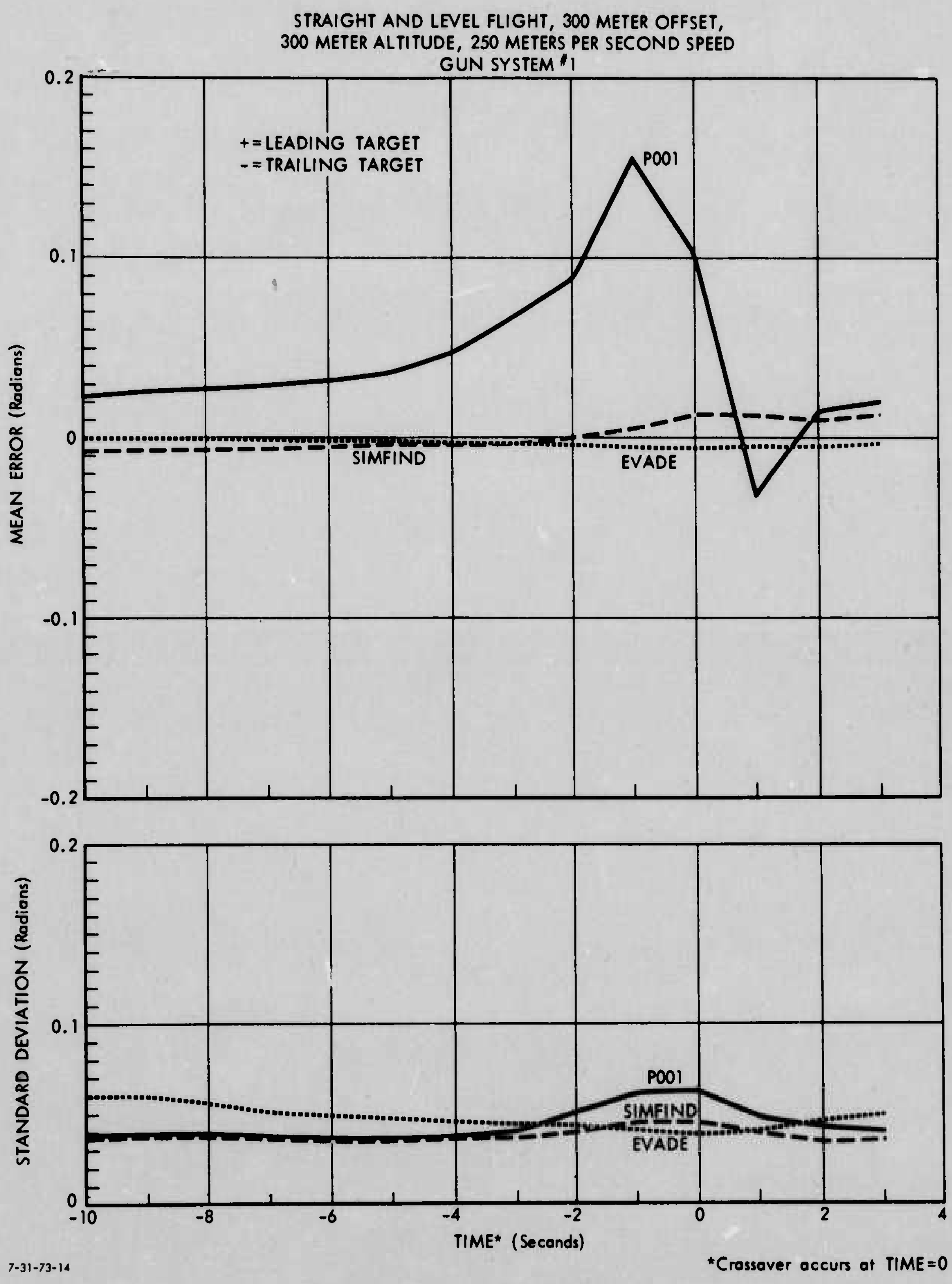

Figure 8. Errors in Gun Azimuth

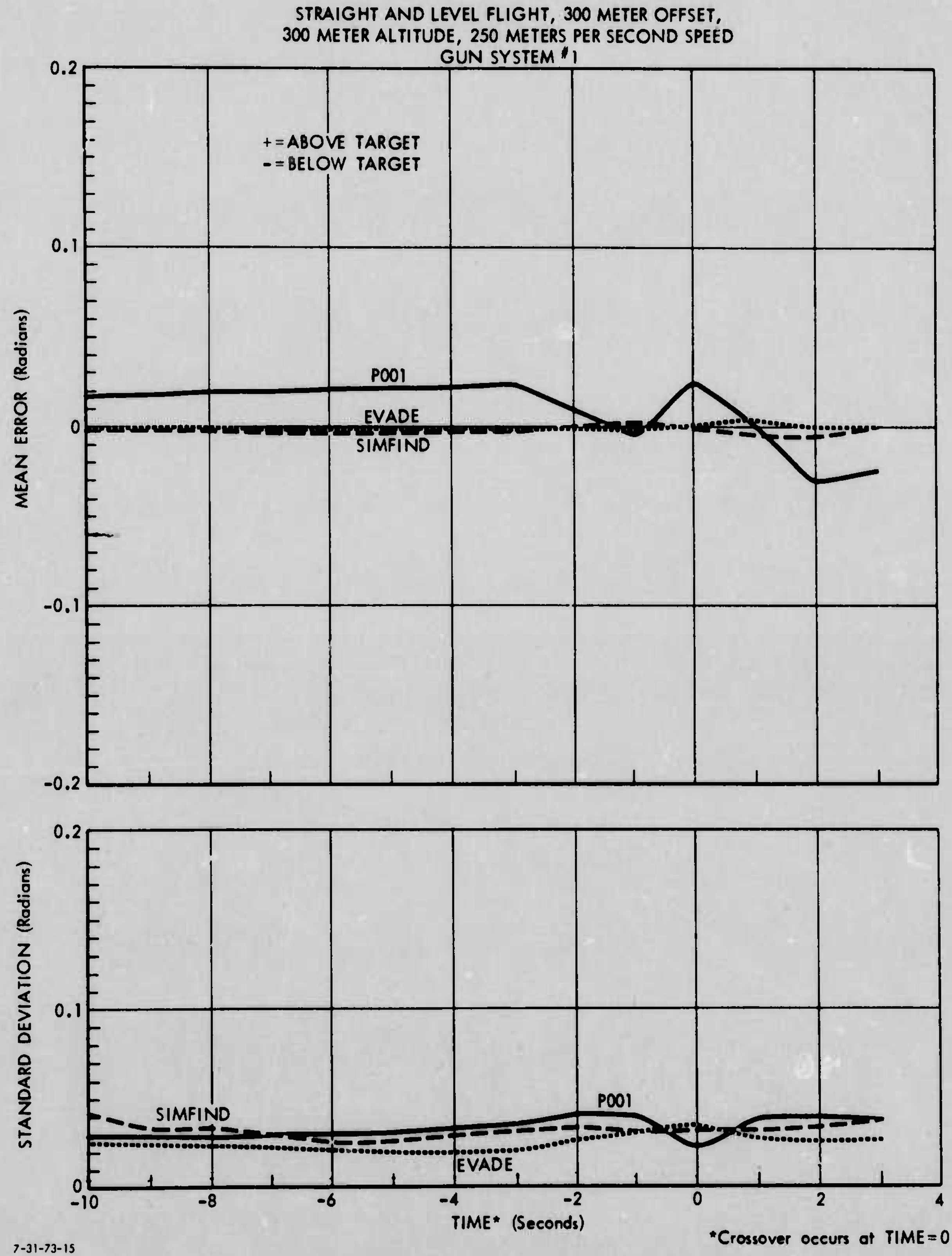

Figure 9. Errors in Gun Elevation

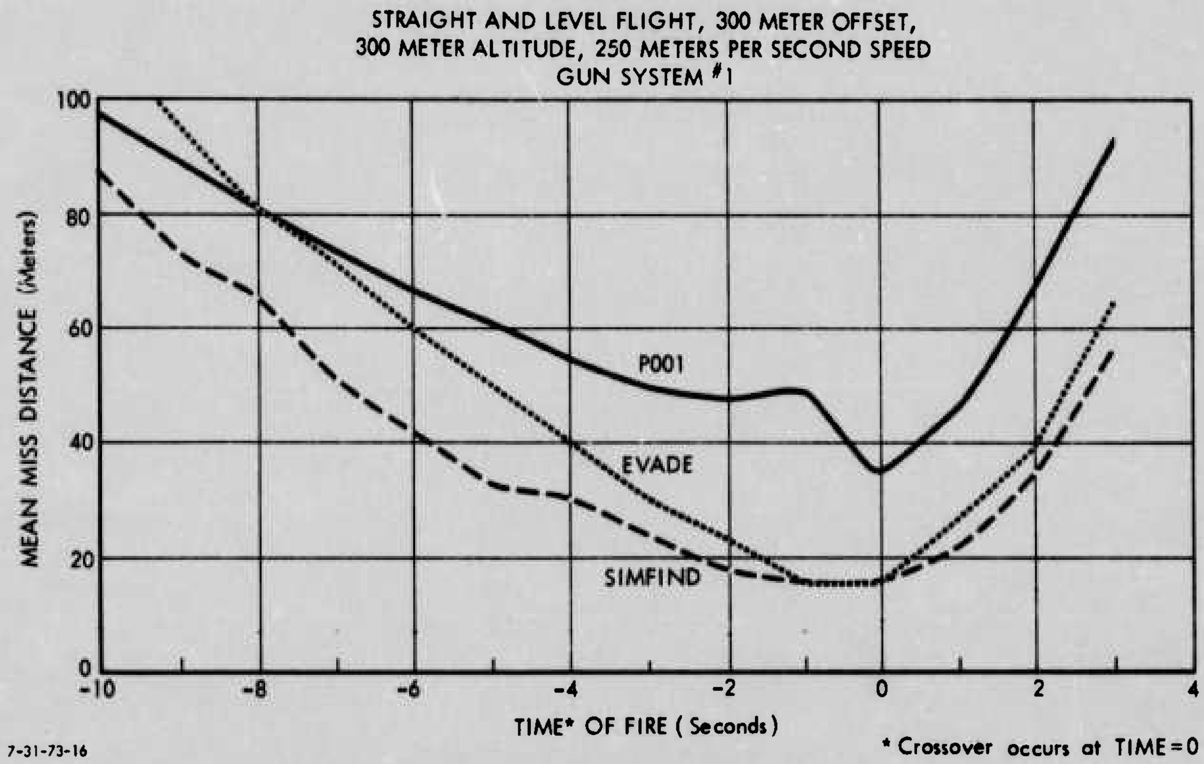

Figure 10. Mean Miss Distance

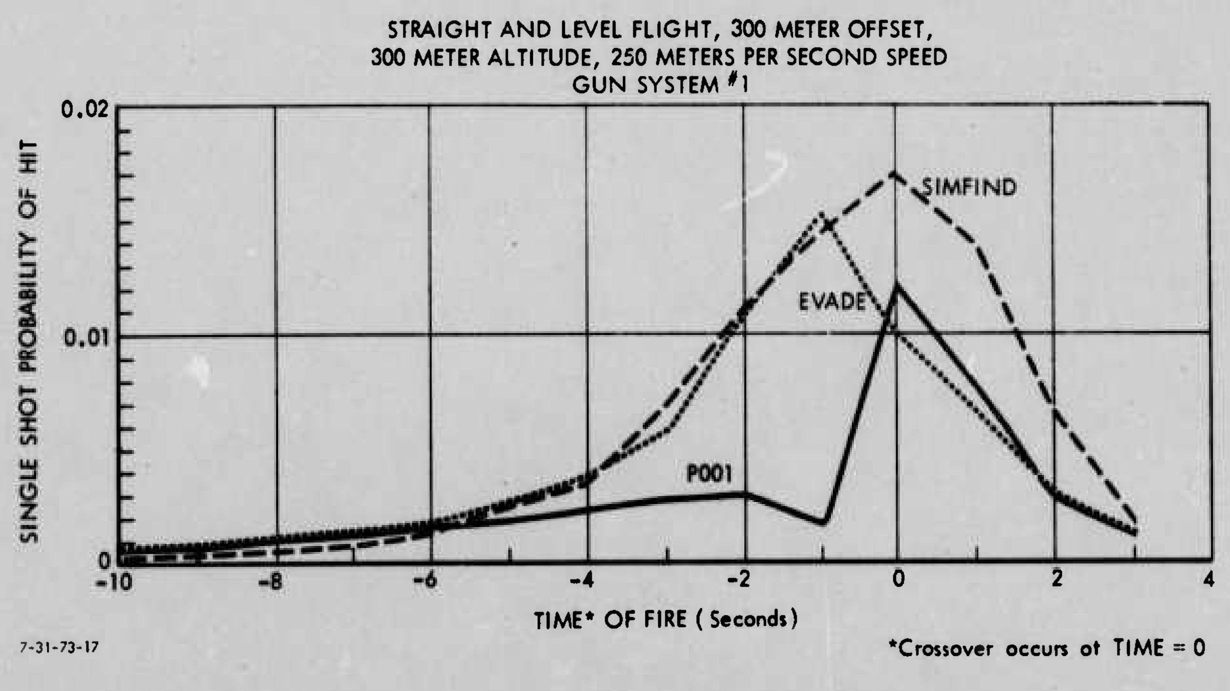

Figure 11. Single Shot Probability of Hit

Chapter V

DETERMINATION OF THE PROJECTILE TRAJECTORY

The material presented in this chapter refers to the models as they existed a year ago. Since that time, both EVADE and SIMFIND have adopted the procedure used in P001.

The models all approximate the projectile trajectory as a straight line (in the direction of fire[11]), with the range from the gun determined as a function of time by the following analytic equation:

$$R = VT/(1 + AT + BT^2)$$

where

R = range
T = time
V = muzzle velocity of projectile
A,B = input constants.

B is equal to zero in both the EVADE and SIMFIND models but is specified by the user in the P001 model.

To examine the accuracy of the above approximation, an investigation was carried out whereby the results obtained therewith were compared with those obtained using a detailed trajectory model[12] based on integration of the equations of motion. In this investigation, the value of input A (and that of input B for P001) was determined by minimizing the error in the approximation when the elevation angle of the gun at fire was 45 degrees. The results, shown in Figures 12 through 15, indicate the P001 approximation to be (1) superior to that of EVADE and SIMFIND and (2) generally within the WSEG Report 190 requirement that the error in range at any given time be no greater than about 0.4 percent (see page 7, footnote 7).[13]

11. The models generally ignore the effect of gravity, both in the determination of the gun firing angles and in the direction of motion of the projectile. The error introduced by this procedure is negligible if the point of closest approach is in fact the aimpoint. When it is not, the hit probability is usually very small, so the error is unimportant. Nevertheless, it is not difficult to introduce an additional correction term due to gravity when the latter case occurs. Such a procedure is already carried out in the SIMFIND model and would be relatively easy to incorporate into the other two models.

12. The model is described in *Equations of Motion for a Modified Point Mass Trajectory*, Ballistics Research Laboratory Report No. 1314, March 1966.

13. It should be pointed out that EVADE currently allows the user the option of using detailed projectile trajectory tables. Thus, if the user has the necessary data available and does not object to the additional computer running time involved, he may use this option to achieve a greater accuracy than is possible by the use of the simple analytic equation. Such tables could also be incorporated relatively easily into the SIMFIND and P001 models if it were felt they were needed.

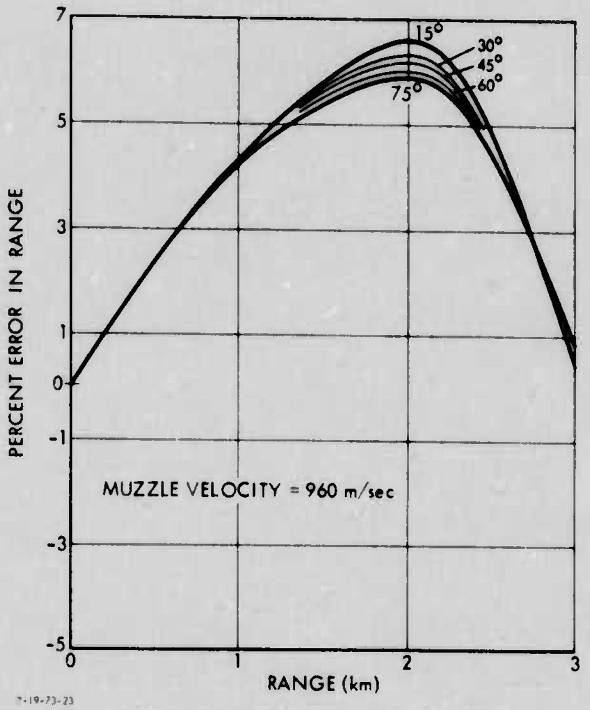

Figure 12. Error in Projectile Trajectory Approximation— EVADE/SIMFIND Equation (23 mm)

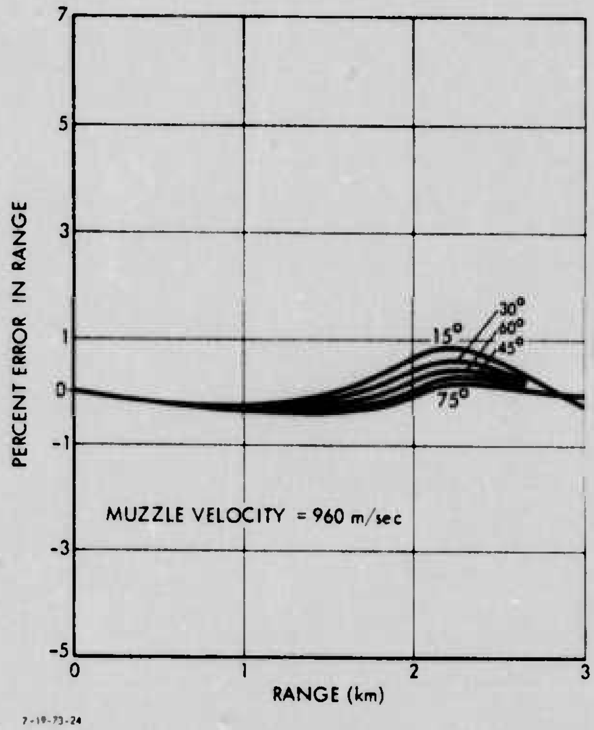

Figure 13. Error in Projectile Trajectory Approximation— P001 Equation (23mm)

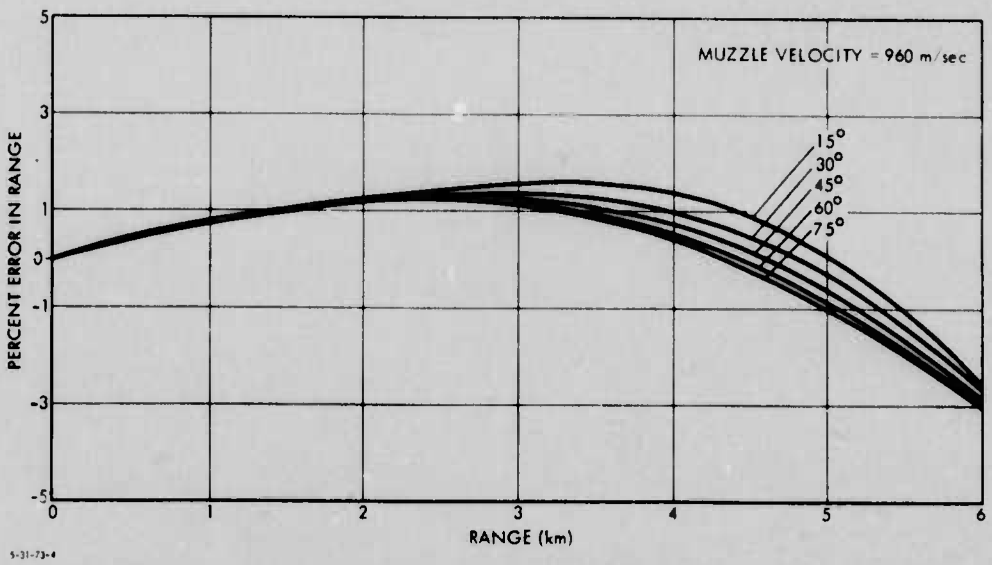

Figure 14. Error in Projectile Trajectory Approximation— EVADE/SIMFIND Equation (57 mm)

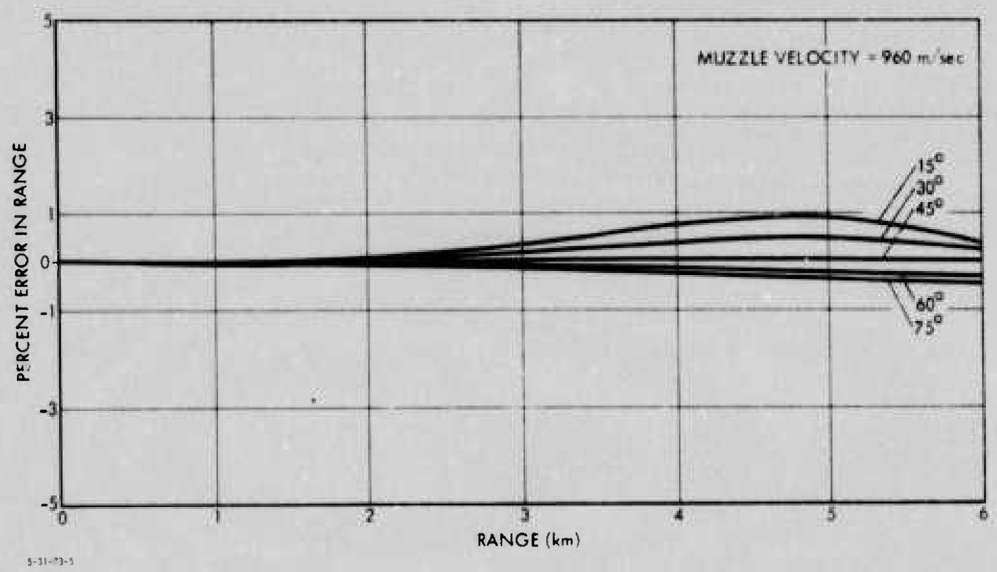

Figure 15. Error in Projectile Trajectory Approximation–P001 Equation (57 mm)

Chapter VI

CALCULATION OF THE SINGLE-SHOT PROBABILITY OF HIT

All three models use the same approximation to evaluate the single-shot probability of hit once the presented area (A) of the aircraft, the dispersion (σ) associated with the projectile, and the mean miss distance (R) have been determined.[14,15] This approximation (the Carlton diffuse target approximation) proceeds as follows. Consider first the case where there is no dispersion. Define a three-dimensional Cartesian coordinate system in which the origin is at the center of the target aircraft, and the x-axis points in the direction of the velocity vector of the projectile at the moment of closest approach to the target. The position of the projectile at impact (or miss) is thus some point (x,y,0) in the x-y plane, and the probability of hit $P_H(x,y)$ is either zero or one, depending on the particular values of x and y, and on the orientation of the target aircraft. In the Carlton diffuse target approximation, $P_H(x,y)$ is taken to be equal to $\exp[-\pi(x^2 + y^2)/A]$, rather than zero or one.

Thus far, the discussion has been for the case where there is no dispersion. In general the dispersion is nonzero, and one computes the probability of hit in this case by multiplying the above expression for $P_H(x,y)$ by the appropriate probability distribution (invariably assumed to be bivariate normal) and then integrating the result over the entire x-y plane.

The approximation is quite good if σ is large compared to the dimensions of the aircraft. Thus, in general, one considers the value of $A/\pi\sigma^2$. When this parameter is small compared to one (as it usually is in practice), the approximation is quite adequate. To examine further the effectiveness of this approximation, an investigation was carried out comparing the probabilities of hit given by the approximation with those obtained using a very detailed representation[16] of the F-4 target aircraft (Figure 16). The results of the investigation,[17] which involved

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14. Because dispersion is determined in the frame of reference of the gun (rather than that of the target aircraft), it turns out to be expedient to evaluate the distance between the projectile and the aircraft when both are equidistant from the gun. This distance in general is not equal to the distance at closest approach, although P001 in essence assumes that it is. The error introduced thereby is very small, however, unless the speed of the aircraft is comparable to that of the projectile (in which case the probability of hit is almost always negligible).

15. The investigation of possible correlation effects between successive rounds from weapons with very high rates of fire is beyond the scope of this study.

16. These representations were furnished by Mr. John R. Bok, Naval Missile Center, Point Mugu, California.

17. The true probability of hit was determined from the detailed representations as follows. First, the mean point of impact (or closest approach) was chosen. (Thus, for example, if the projectile path were normal to the (continued on next page)

variations in the direction as well as the magnitude of the mean miss vector, are shown in Figures 17 through 19. As is apparent, the agreement is quite good for $A/\pi\sigma^2 \ll 1$ (0.24 in Figure 17), marginal for $A/\pi\sigma^2 \sim 1$ (1.49 in Figure 18), and quite poor for $A/\pi\sigma^2 \gg 1$ (5.98 in Figure 19). Thus, since in practice $A/\pi\sigma^2$ is almost always less than or equal to 0.25, the conclusion that the approximation used by the models is adequate is seen to be valid.

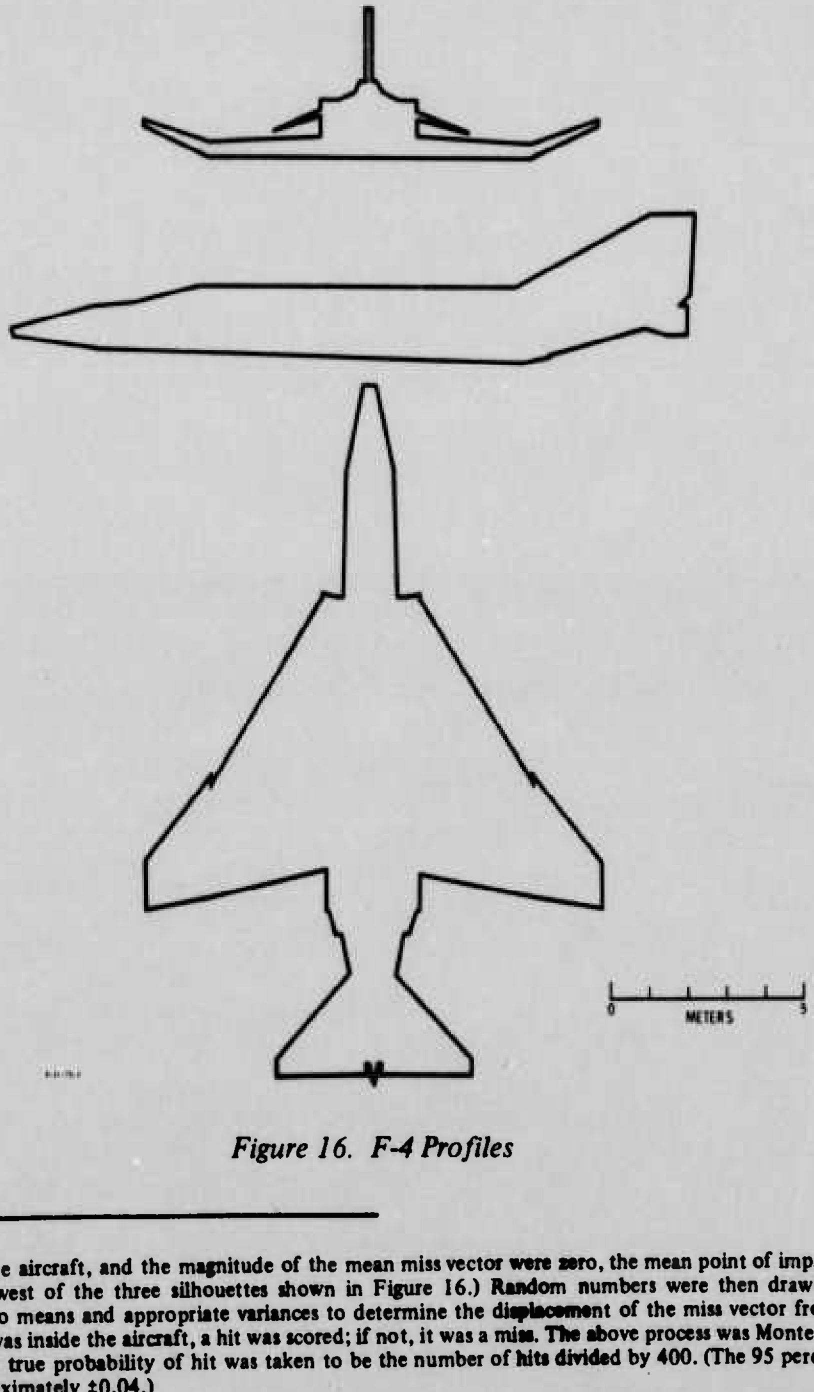

Figure 16. F-4 Profiles

(cont'd) bottom of the aircraft, and the magnitude of the mean miss vector were zero, the mean point of impact would be in the middle of the lowest of the three silhouettes shown in Figure 16.) Random numbers were then drawn from normal distributions with zero means and appropriate variances to determine the displacement of the miss vector from its mean. If this new miss vector was inside the aircraft, a hit was scored; if not, it was a miss. The above process was Monte Carloed a total of 400 times, and the true probability of hit was taken to be the number of hits divided by 400. (The 95 percent confidence limits were thus approximately ±0.04.)

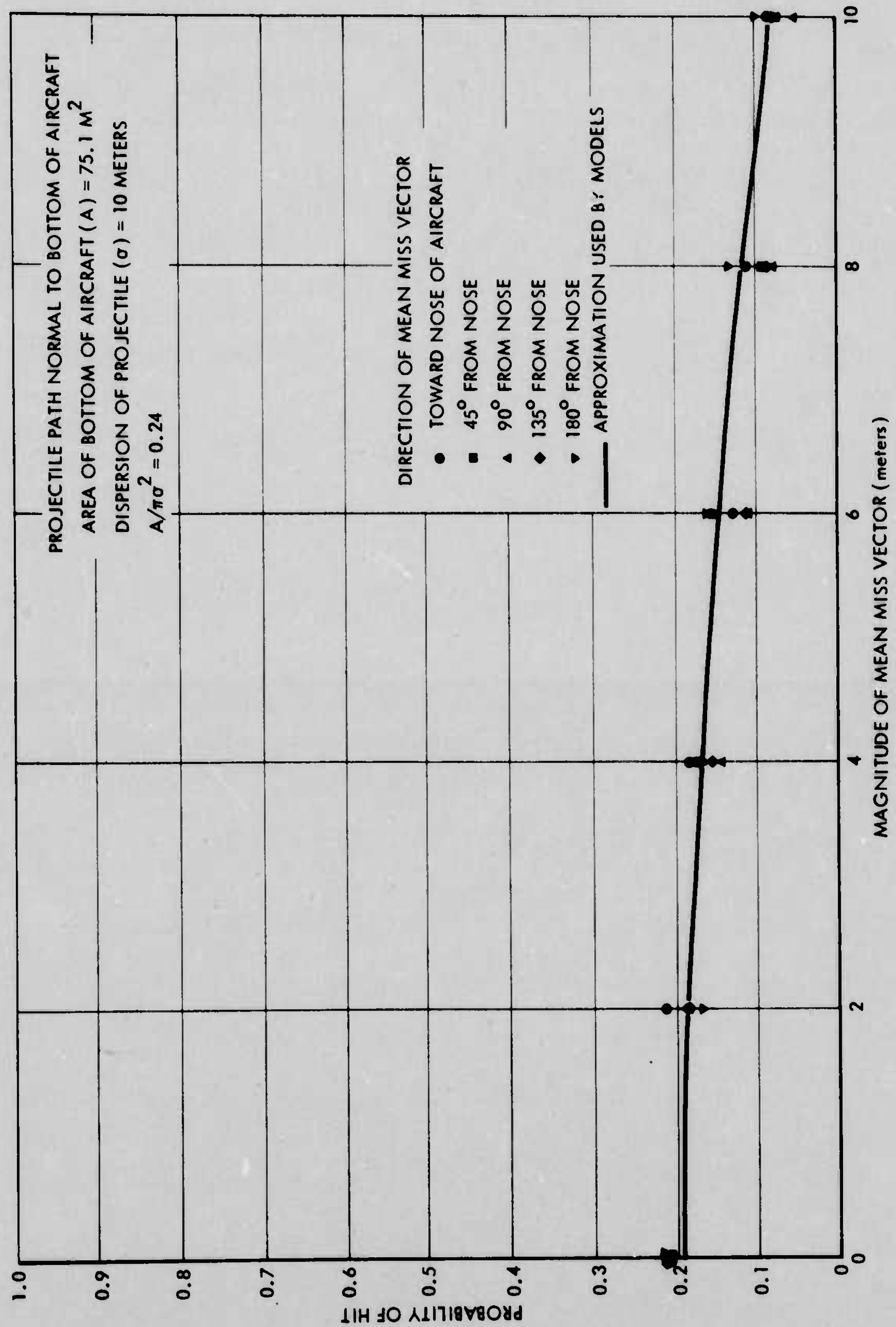

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Figure 17. Probability of Hit Investigation–Projectile Dispersion = 10 Meters

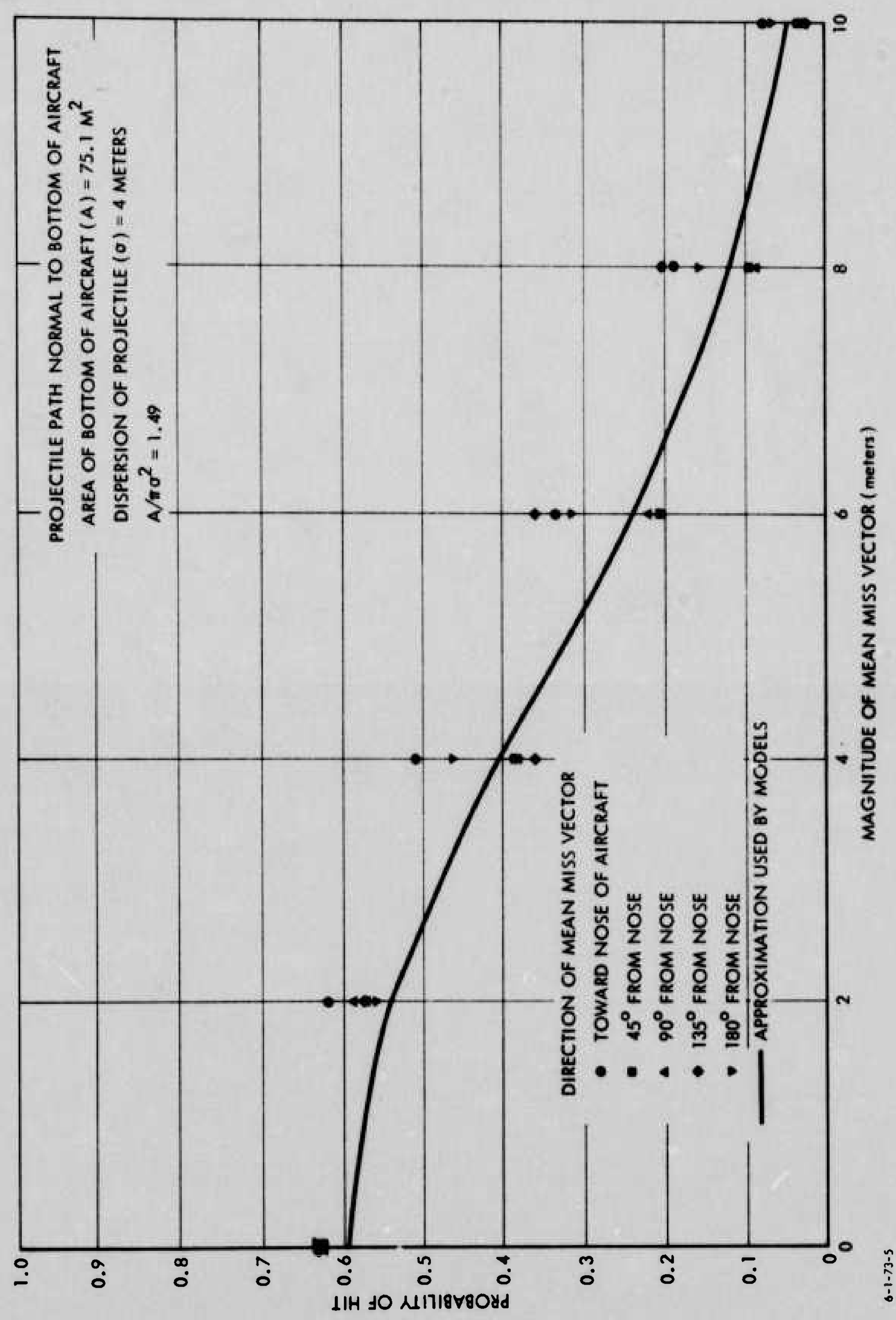

Figure 18. Probability of Hit Investigation—Projectile Dispersion = 4 Meters

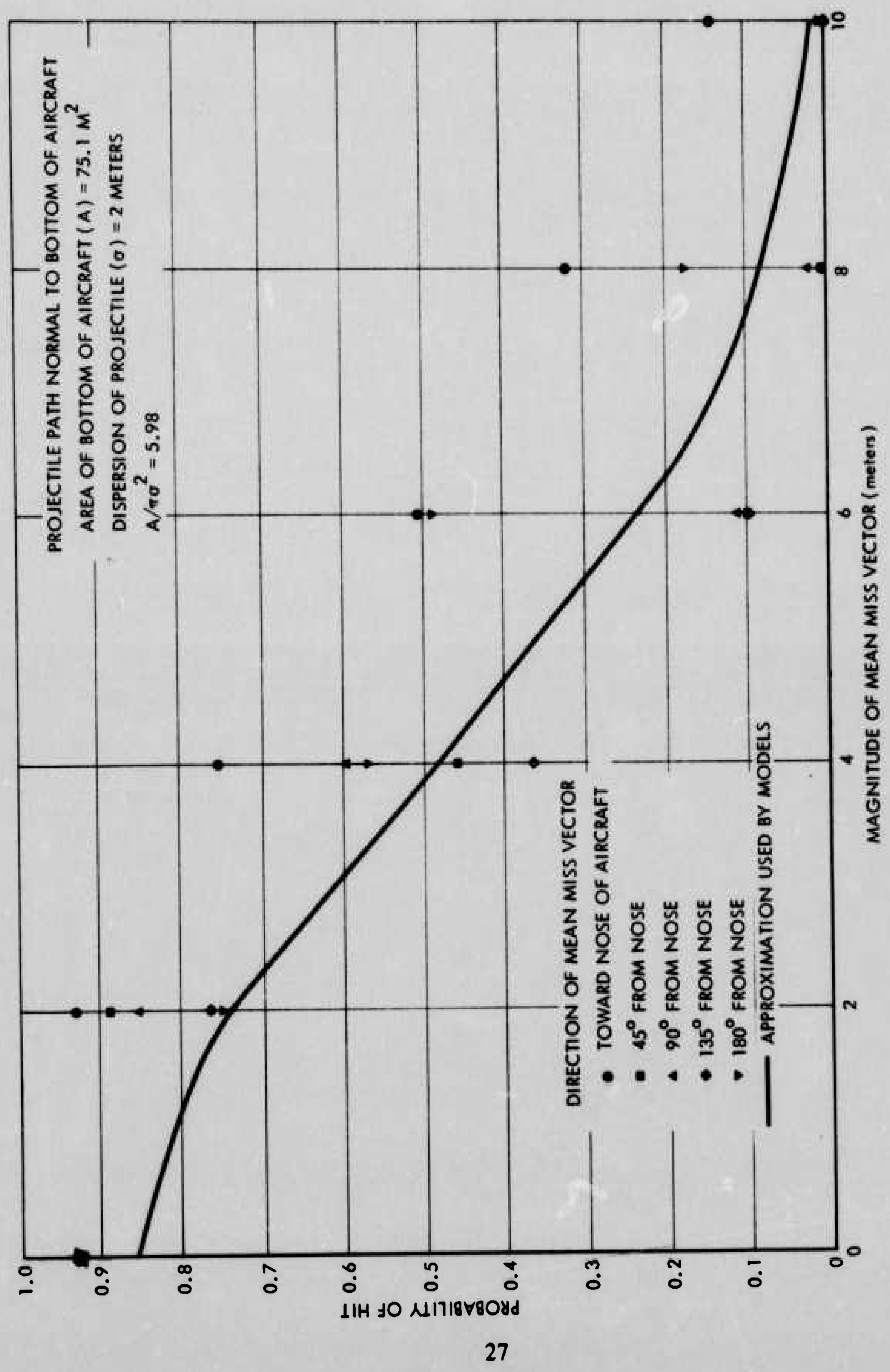

Figure 19. Probability of Hit Investigation—Projectile Dispersion = 2 Meters